DISPERSION OF TRANSIENT SIGNALS IN MICROSTRIP TRANSMISSION LINES

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ABSTRACT

The distortion of an electrical pulse caused by dispersion as it propagates along a microstrip line is investigated. A model for dispersion of the phase constant is selected to meet the frequency, accuracy, and microstrip parametric requirements. Numerical integration and Taylor series expansion techniques are used to compute the shape of the DC dispersed pulses having square and Gaussian envelope shapes. Taylor series expansion methods are more convenient for the analysis of RF pulses.

I. INTRODUCTION

The design of MIC's requires a knowledge of switching and transient signal behavior in microstrip transmission lines and semiconductor structures. The distortion of DC and RF pulses in waveguides and dispersive material has received in the past considerable attention [1]-[3]. However, distortion of pulses, both DC and RF, in microstrip lines has not yet been examined thoroughly [4].

As an electrical pulse travels along a microstrip line it becomes distorted due to the dispersion and attenuation characteristics of the line. While the electric and magnetic fields are confined to one material in waveguides, coaxial lines and strip lines, the microstrip is open so that the fields are partially in the air and partially in the dielectric. The air-dielectric interface prevents propagation of a pure TEM mode. Therefore, the phase constant is not a linear function of frequency, and it results in dispersion.

Below a certain frequency (f_t) the propagation is approximately TEM, and dispersion does not occur. Pulses which have a spectral content above f_t will be dispersed since the higher harmonics of the pulse will travel at a slower velocity than the lower harmonics. This paper combines existing microstrip dispersion formulas and analytical techniques to determine the shape of the DC and RF dispersed pulses having square and Gaussian envelope shapes.

II. DISTORTION OF SIGNALS

The voltage or electric field at z=0 (a reference point on the microstrip line) of a transient waveform is represented by

$$v(t,z=0) = \begin{cases} v(t) & -T/2 \le t \le T/2 \\ 0 & \text{elsewhere} \end{cases} \qquad (1)$$

In the frequency domain, the signal can be written as

$$V(\omega,z=0) = \int_{-T/2}^{T/2} v(t,z=0)e^{-j\omega t}dt \qquad (2)$$

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where v(t) and V(ω) form a transform pair. For certain transient signals, such as a square pulse, the limits $-T/2 \le t \le T/2$ define the pulse width and the signal is confined to a short time period. For a Gaussian pulse, the time range of $-\infty<t<\infty$ is needed to completely characterize the response.

At a distance L, the signal (or pulse) in the frequency domain becomes

$$V(\omega,z=L) = V(\omega,z=0)e^{-\gamma(\omega)L} \qquad (3)$$

The frequency dependent propagation constant is

$$\gamma(\omega) = \alpha(\omega) + j\beta(\omega) \qquad (3a)$$

where $\alpha(\omega)$ and $\beta(\omega)$ are, respectively, the attenuation and phase constants. For this investigation, the frequency dependent attenuation constant $\alpha(\omega)$ is assumed to be negligible so that (3) reduces to

$$V(\omega,z=L) = V(\omega,z=0)e^{-j\beta(\omega)L} \qquad (4)$$

Taking the inverse transform of (4) leads to the time domain representation of the pulse at z=L, and it can be written as

$$V(t,L) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega,z=0)e^{j[\omega t-\beta(\omega)L]}d\omega \qquad (5)$$

For lossless lines the phase constant $\beta(\omega)$ can be written as

$$\beta(\omega) = \omega\sqrt{\mu\varepsilon(\omega)} = \frac{\omega}{c}\sqrt{\varepsilon_{r_{eff}}(\omega)} \qquad (6)$$

The expression for V(ω,z=0), the transform of v(t,0), is easily obtained for many common wave shapes such as square, Gaussian, triangular pulses and any RF pulse modulated by these waveforms. The transforms of more complex waveforms are constructed using these basic waveforms.

III. FREQUENCY DEPENDENT PHASE CONSTANT

Numerous methods have been used to determine $\varepsilon_{r_{eff}}(\omega)$ for microstrip lines. Many papers use full wave solutions such as the spectral domain [5] or transverse current distribution methods [6]. However, these methods depend on time consuming computations and not on closed form equations which would be most desirable when confronted with the evaluation of (5). Some papers have curve fitted equations for $\varepsilon_{r_{eff}}(\omega)$ which are simple to use. However, none of these equations extend above 20 GHz, and they are not adequate for many transient signals that have frequency components up to 100-200 GHz.

Two methods that may be used to calculate $\varepsilon_{r_{eff}}(\omega)$ which provide physical insight and fairly simple closed form expressions, although they may not be as accurate as the full wave analysis, are

1) Coupled Modes (TEM, TE, and TM modes)[7].
2) Single Longitudinal Section Electric(LSE)[8].

Equations which use coupled modes are given by Schneider (TEM/TE) [9], Carlin (TE/TM) [10], Kobayashi (TEM/TM) [11], Pramanick and Bhartia (TEM/TE) [12], and Yamashita (curve fitting using the TE mode) [13]. Getsinger [8] uses the LSE model to determine the frequency dependent dielectric constant. From the standpoint of analytical rigor, simplicity, and agreement with other existing data it was found that the model for $\varepsilon_{r_{eff}}(\omega)$ of Pramanick and Bhartia [12] was as accurate as any of the others. For this model, $\varepsilon_{r_{eff}}(\omega)$ is expressed as

$$\varepsilon_{r_{eff}}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{r_{eff}}(0)}{1 + (f/f_t)^2} \quad (7)$$

$$\frac{1}{f_t^2} = \frac{4\mu_0^2 h^2 \varepsilon_{r_{eff}}(0)}{Z_0^2 \varepsilon_r} \quad (7a)$$

where ε_r = dielectric constant of the substrate
$\varepsilon_{r_{eff}}(0)$ = effective dielectric constant at zero frequency
h = height of the substrate
Z_0 = characteristic impedance of the line

IV. EVALUATION OF INTEGRAL EQUATION

Two different methods to evaluate the integral equation of (5) are examined in this section. The complexity of the frequency dependent phase constant $\beta(\omega)$ precludes solving for the integral in closed form. Thus numerical integration techniques, as well as a quadratic approximation (Taylor series expansion method) are used to evaluate the integral of (5).

Numerical integration is the most straight forward technique for evaluating (5), but its accuracy depends on the amount of computer time and storage space available. DC pulses use less computer resources than RF pulses, and this method is best suited for them. The Taylor series expansion method [1] is an approximation to the full integration of (5). Even though it is slightly less accurate than numerical integration, it provides data which compare well with those obtained from numerical integration; however, it requires much less computer time to evaluate the integral, especially for RF pulses.

In (5) the limits of integration are $-\infty<\omega<\infty$; however, beyond a certain radian frequency, ω_L, the contributions to the integral are negligible. Narrower pulses have a higher frequency content and will need a higher ω_L; thus significant parts of the integral are not excluded. If τ is the width of the pulse, then

$$\omega_L = \xi/\tau \quad (8)$$

where ξ is a constant which depends on the waveshape. For example, a square pulse with a sharp rise time (high frequency content), ξ is about 500. For a Gaussian pulse with a slower rise time a ξ of 20 is sufficient. Thus, (5) becomes

$$v(t,L) \simeq \frac{1}{2\pi}\int_{-\xi/t}^{\xi/t} V(\omega,z=0)e^{j[\omega t-\beta(\omega)L]}d\omega \quad (9)$$

This can be written as a series approximation of the form

$$v(t,L) \simeq \frac{1}{2\pi}\sum_{i=1}^{N} V(\omega_i,z=0)e^{j[\omega_i t-\beta(\omega_i)L]}\Delta\omega_i \quad (10)$$

where N = the number of divisions in the frequency spectrum

$\Delta\omega_i = \frac{2\xi/\tau}{N}$ = the width of each uniform segment

Since we are concerned only with the real part of the pulse, (10) becomes

$$v(t,L) \simeq \frac{1}{2\pi}\sum_{i=1}^{N} V(\omega_i,z=0)\cos[\omega_i t-\beta(\omega_i)L]\Delta\omega_i \quad (11)$$

Equation (11) is easily programmed on the computer once $V(\omega_i,z=0)$, the Fourier transform of the pulse being considered, is known.

In addition to using numerical integration to evaluate (5), there are approximate methods which can represent it in closed form. One such method is the Taylor series expansion (also referred to as quadratic approximation) where the phase constant $\beta(\omega)$ is approximated in the vicinity of ω_0 by the first three terms of the Taylor series expansion. For the cases being investigated, it is a good assumption to consider the phase constant to be a quadratic function of frequency. If the pulses are sufficiently wide compared to the modulating frequency, then only a small segment of the $\beta(\omega)$ curve is used and such an approximation is quite valid.

It is possible to obtain closed form solutions to (5) if the signal in the frequency domain at z=0, $V(\omega,z=0)$, can be written in closed form. Closed form expressions have been derived to evaluate (5) for Gaussian [1] and square modulated pulses [2], [3] as they are dispersed while they travel in a waveguide. These expressions were modified for microstrips where the frequency dependent $\beta(\omega)$ of (6) was formed using Pramanick and Bhartia's model [12] for $\varepsilon_{r_{eff}}(\omega)$.

V. COMPUTATIONS

To verify and compare the different models and methods, a number of measurements were made. Only a representative sample of them will be presented here.

The variations of $\varepsilon_{r_{eff}}(\omega)$ as a function of frequency for six different models are shown in Figure 1. These are representative for a microstrip with a dielectric constant of ε_r=10.2, and with a width of w=0.020" and a height of h=0.025".

The distortion of a τ=10 psec(3 dB width) DC Gaussian pulse traveling a distance of L=0.354" along this microstrip line, as predicted using four of these models, is displayed in Figure 2. For comparison, the undistorted pulse is also exhibited in the figure. The position of the undistorted pulse has been determined assuming its velocity of propagation is based on the effective dielectric constant of the microstrip at zero frequency. The waveforms of the distorted pulses were computed using numerical integration for the evaluation of (5), as outlined in Section IV. Based on the results of Figure 2 and 3, as well as other computations and comparisons, it was decided to use Pramanick and Bhartia's model for $\varepsilon_{r_{eff}}(\omega)$ for the continuation of the investigation of pulse distortion due to dispersion.

The dispersion of a DC square pulse of width τ=250 psec traveling a distance of L=1" along a microstrip with ε_r=10.2, w=0.025", h=0.150" is shown in Figure 3. It is apparent that major distortion peaks along the leading and trailing edges of the pulse have been created. This distortion pulse was also obtained using numerical integration.

To compare the validity of the Taylor series approximation method, as outlined in Section IV, computations were made for the envelope of distorted Gaussian and square RF pulses using the Taylor series expansion method. The results are displayed, respectively, in Figures 4 and 5 where they are compared with the computations of the complete RF pulses obtained using numerical integration. It is evident from these and other computations not included here that the Taylor series expansion method yields good waveform approximations to the distorted RF pulses with a considerable reduction in computation time.

VI. CONCLUSIONS

The distortion of DC and RF pulses as they propagate along a microstrip line was investigated using dispersion models in conjunction with numerical integration and Taylor series expansion approximation techniques. Pramanick and Bhartia's dispersion model provided a convenient closed form expression to evaluate the distortion of a pulse propagating along a microstrip line. Numerical integration was required to analyze DC pulses. RF pulses can be analyzed using either numerical integration or the Taylor series expansion method, with only a slight decrease in accuracy but considerable improvement in computational efficiency.

VII. ACKNOWLEDGEMENT

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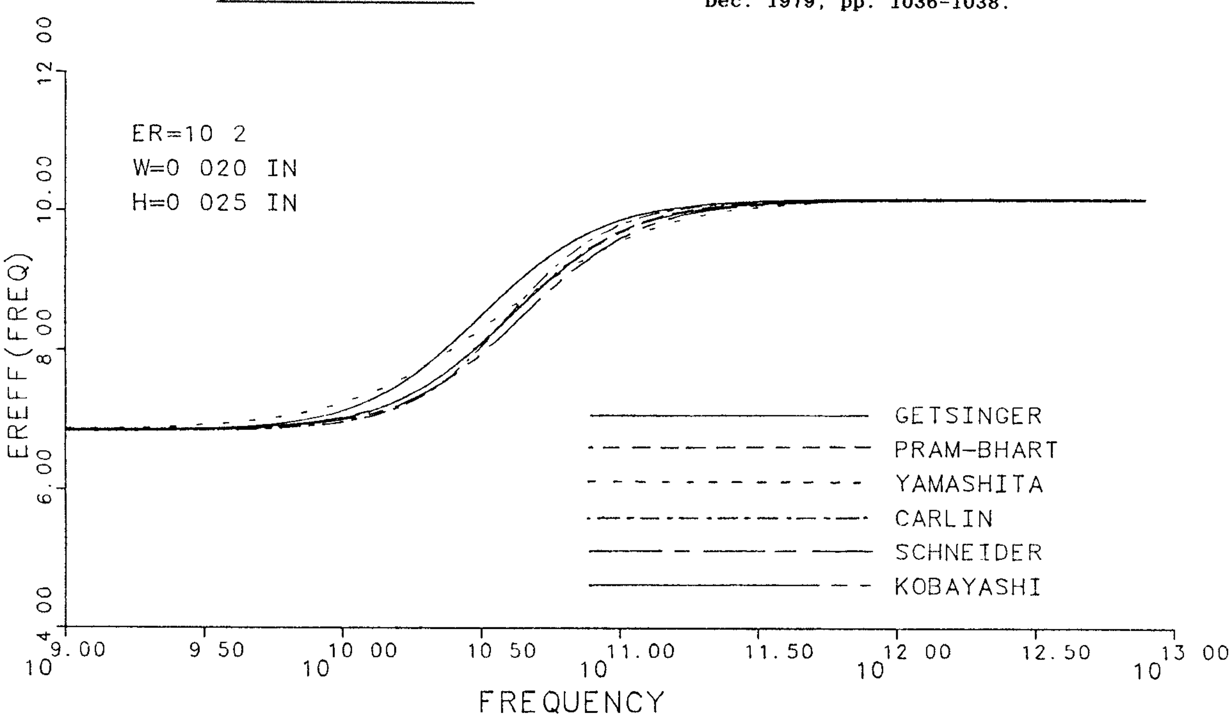

Fig. 1. Effective dielectric constant of a microstrip line as a function of frequency for different proposed models (ε_r= 10.2; w/h<1.0).

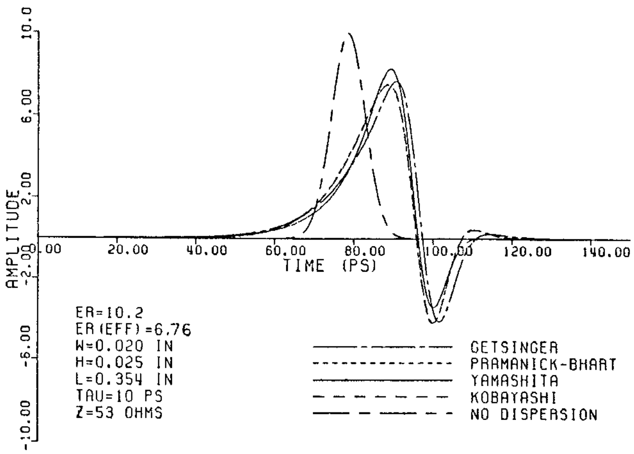

Fig. 2. Gaussian DC pulse dispersion at a distance L in a microstrip line using different proposed models for $\varepsilon_{r_{eff}}(\omega)$ (ε_r = 10.2; w/h<1.0).

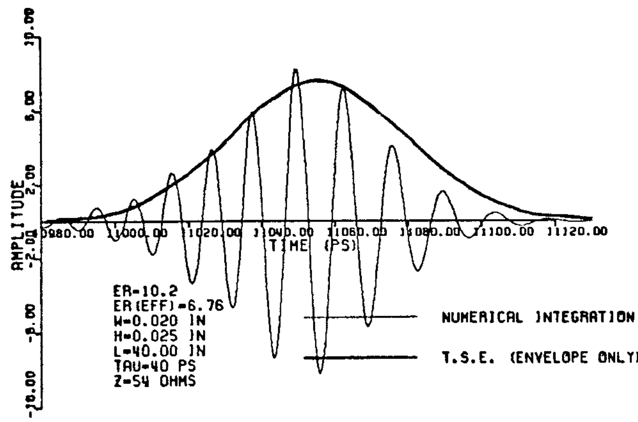

Fig. 4. Gaussian RF pulse dispersion computed using numerical integration for the full modulated pulse and Taylor series expansion for its envelope (carrier frequency f_0 = 75 GHz).

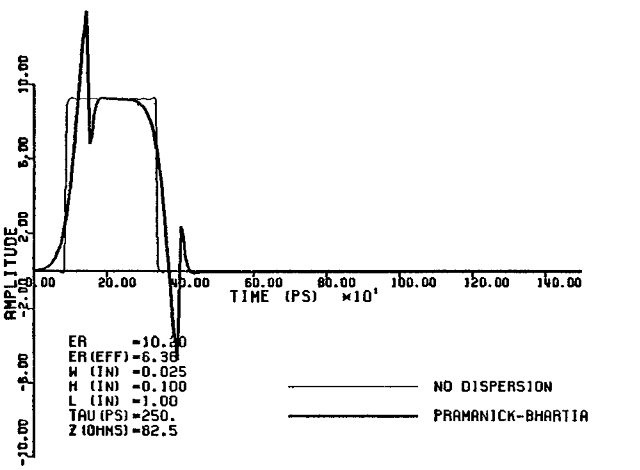

Fig. 3. Square DC pulse dispersion at a distance L in a microstrip line computed using numerical integration and Pramanick and Bhartia's dispersion model.

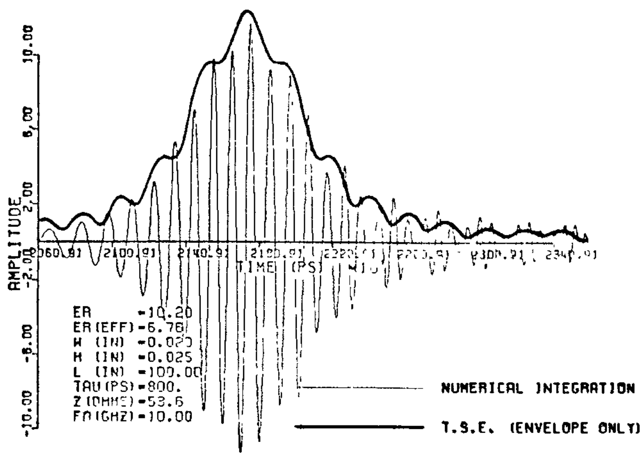

Fig. 5. Square RF pulse dispersion computed using numerical integration for the full modulated pulse and Taylor series expansion for its envelope (carrier frequency f_0 = 10 GHz).